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EXPLORING THE EXPLANATION OF PRE-SERVICE TEACHER IN MATHEMATICS TEACHING PRACTICE

Wasilatul Murtafiah^{1,2}, Cholis Sa'dijah², Tjang Daniel Candra², Susiswo², Abdur Rahman As'ari²

¹Universitas PGRI Madiun, Jalan Setiabudi 85, Madiun, Indonesia

²Universitas Negeri Malang, Jalan Semarang 5, Malang, Indonesia

Email: cholis.sadiah.fmipa@um.ac.id

Abstract

This study aims to describe the types of explanations made by pre-service teachers in mathematics learning. In this research, the types of explanations are used to describe the explanatory trends used by pre-service teachers in mathematics teaching. The descriptive qualitative research was chosen in this research. The research subjects are pre-service teacher as the students of Mathematics Education of PGRI Madiun University and Madura University who are studying Field Experience Practice. Of the 105 mathematics student, five students with a cumulative grade achievement of more than 3.50 were observed during the teaching practice at the school for approximately five meetings. The research data was obtained from observation, video recording, and interview. Data analysis was done through data condensation, data presentation, and conclusion/verification focused on pre-service teacher explanation on mathematics learning activity. The research findings indicate that the explanation used by the pre-service teacher in the mathematics learning starting from the most frequently used is the descriptive explanation (51,7%), giving of reason (36,2%) and interpretative (12,1%). Descriptive explanations are used to describe mathematical procedures. The type of reason-giving explanation is used to explain reasons based on mathematical principles. Furthermore, the interpretative explanation is used to explain the concepts and facts of mathematics.

Keywords: Explanation, Pre-service Teacher, Mathematics Teaching.

Abstrak

Penelitian ini bertujuan untuk mendeskripsikan jenis penjelasan yang dilakukan oleh mahasiswa calon guru dalam pembelajaran matematika. Jenis-jenis penjelasan digunakan untuk mendeskripsikan kecenderungan penjelasan yang digunakan mahasiswa calon guru dalam membelajarkan matematika. Penelitian kualitatif deskriptif dipilih dalam penelitian ini. Subjek penelitian adalah mahasiswa Pendidikan Matematika Universitas PGRI Madiun dan Universitas Madura yang sedang menempuh mata kuliah Praktek Pengalaman Lapangan. Dari 105 mahasiswa calon guru matematika, dipilih sebanyak 5 mahasiswa calon guru dengan indeks prestasi kumulatif lebih dari 3,50 untuk diobservasi pada saat praktek mengajar di sekolah selama kurang lebih 5 pertemuan. Data penelitian diperoleh dari observasi, rekaman video dan wawancara. Data dalam penelitian ini dianalisis melalui kondensasi data, penyajian data, dan penarikan kesimpulan/verifikasi yang difokuskan pada penjelasan mahasiswa calon guru pada kegiatan pembelajaran matematika. Temuan penelitian menunjukkan bahwa penjelasan yang digunakan mahasiswa calon guru pada pembelajaran matematika mulai dari yang paling sering digunakan adalah jenis penjelasan deskriptif (51,7%), pemberian alasan (36,2%) dan interpretatif (12,1%). Penjelasan deskriptif digunakan untuk menjelaskan prosedur matematika. Jenis penjelasan pemberian alasan digunakan untuk menjelaskan alasan berdasarkan prinsip matematika. Sedangkan penjelasan interpretatif digunakan untuk menjelaskan konsep dan fakta matematika.

Kata kunci: Penjelasan, Mahasiswa Calon Guru, Pembelajaran Matematika

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Mathematics learning is compulsory learning in schools from elementary to high school. Through mathematics learning students are equipped with mathematical materials in the form of facts, concepts, procedures and mathematical principles. At the time of learning, teachers provide explanations to understand mathematics material to students. Explanations are always conducted on various mathematical learning, e.g. explanations

based on mathematical contexts (e.g., geometry, arithmetic, algebra), types of learning activities that occur (e.g., concepts, procedures, guesswork), the purpose of explanation (e.g., explanation of a particular case, explanations leading to generalization), and the age level of students (Levenson, Tsamir, & Tirosh, 2010).

To provide an explanation that can understand mathematical concepts to students, math teachers must have good communication skills. Given that supporting the understanding of student mathematics is an essential effort of teachers in learning mathematics (Lachner & Nückles, 2016). Students should gain an in-depth understanding of the rules and concepts of mathematics to transfer and generalize their knowledge flexibly on other tasks (Richland, Stigler, & Holyoak, 2012; Schoenfeld, 1988). In learning mathematics in Indonesia, teachers provide explanations to understand students, especially on new material. However, previous research has shown that instructional explanations do not contribute significantly to meaningful learning since they involve students in constructing their understanding of learning activities (Schworm & Renkl, 2006; Wittwer & Renkl, 2008). However, the findings do not indicate that instructional explanations are considered ineffective since their effectiveness is highly dependent on the design and quality of teachers in designing learning (Wittwer & Renkl, 2008).

Other studies have emphasized the primary role teachers play in effective learning (Baumert et al., 2010; Kunter et al., 2013). It is suspected that pedagogical content knowledge, in particular, can assist teachers and pre-service teachers in generating clarity with high-level clarity, since pedagogical content knowledge typically consists of content-specific strategies on ways to create real lesson material for students (Ball, Thames, & Phelps, 2008; Baumert et al., 2010). In-depth content knowledge is an essential prerequisite to providing instructional explanations that effective in mathematics for producing explanations with high-level process orientation, textual features that serve as scaffolds for meaningful mathematical constructs for students (Lachner & Nückles, 2016).

Providing good instructional clarification is also essential for student learning as suggested by research indicating that obscure and incomplete teacher explanations may disrupt student learning (Borko et al., 1992). Because explanation remains an important mathematics curriculum component, the explanation in the mathematics class is seen as an important skill form to be analyzed and understood (Perry, 2000). The further research should be done to find out why teachers and pre-service teachers provide less process-oriented explanations than are desired in student learning (Lachner, Jarodzka, & Nückles, 2016). Through the explanation given by pre-service teachers to students at the time of classroom, learning practice will be described type, purpose and purpose of explanation given to the learning of mathematics.

The explanation is an attempt to give an understanding of something to others (Hargie, 2006). In the context of education, a good explanation in teaching is essential to open students' material understanding. Explanations are used to enhance teaching by enabling and integrating existing and new knowledge (Jeong & Chi, 2007). Explains supporting teaching to fill the missing information, integrating information in learning materials, integrating new information with prior knowledge, and monitoring false knowledge (Jeong & Chi, 2007). While generating an explanation improves learning, the quality of the explanation is tailored to the student's prior knowledge (Chi, 2009; Jeong & Chi, 2007).

In mathematics learning, the form of explanation may differ depending on the class habits. (Levenson

et al., 2010). In school math learning, students are usually involved in the practice of carrying out a prescribed procedure or the following instruction; an explanation can consist of figure out the steps of the procedure used (Yackel, Cobb, & Wood, 1999). Explanations can explain how to do something and why to do something (Perry 2000). Explanation of learning aims to explain concepts, procedures, events, ideas and class issues to help students understand, learn and flexible use information (Leinhardt, 1990). Explanations of learning can be characterized more specifically, including (a) learning objectives related to the use of explanations; (b) the use of explanations on learning as the primary teaching strategy or as complementary to other teaching strategies; and (c) the use of instructional explanations in instructional dialogue (Wittwer & Renkl, 2008). There are four primary variables consistently linked to a teacher's ability to explain terminology, concepts, and processes more efficiently: orientation, key usage, summary, and communication skills (Odora, 2014).

Based on the previous research result, there are some ways of classifying the explanations in the mathematics class. There is an explanation of conceptual calculations and explanations, in which the calculation description describes a process, procedure, or steps taken to solve a problem whereas conceptual explanation explains the reasons for the steps, which relate the procedure to the student's conceptual knowledge (Fuchs et al., 1997). The next form of explanation is a general explanation beyond the specific problem and the specific explanation for the problem (Perry, 2000). There are also formal and informal explanations, in which formal explanations are characterized by strict arguments according to mathematical definitions and theorems while informal explanations consist of illogical arguments, hunches, and intuitions (Raman, 2002). Another form of explanation in mathematics learning is a formal and concrete explanation, a concrete explanation using real-world experiences to give meaning to mathematical expressions and formal explanations using only mathematical definitions and theorems (Tsamir & Sheffer, 2000).

Furthermore, another form of explanation is a mathematical and practice based explanation, in which mathematical-based explanations are according to mathematical definitions or previously learned mathematical properties, and often use mathematical reasoning suitable for elementary school students. While the explanation of practice is an explanation that depends not only on mathematical concepts but using visual or manipulative aids and based on real-life context (Levenson et al., 2010). The type of explanation that has not been studied in mathematics learning is the type of explanation described in Table 1.

Table 1. Type of Explanation (Hargie, 2006)

Type of Explanation	Definition
Interpretative explanations	The interpretive explanation answers the question, 'What?' Interpret or clarify a problem or determine the central meaning of a term or statement.
Descriptive explanations	A descriptive explanation answers the question, 'How?' This description describes processes, structures, and procedures.
Reason-giving explanations	An explanation that gives the reason for answering the question, 'Why?' Involves reasons based on principles or generalizations, motives, obligations, or values. Included in the explanation of reasoning is a decision based on cause and function (Pavitt, 2000).

This type of interpretive, descriptive and reasoning explanation is thought to have appeared in mathematics learning. The classification of this type of explanation is certainly more complete to explore mathematical objects (facts, concepts, procedures and principles of mathematics) when compared with the types of explanations that have been studied previously. In this research, the researcher used the definition of each type of explanation in Table 1 to describe the explanation type of pre-service teacher and how much the tendency of its use in learning mathematics that includes explanations of mathematical objects.

METHOD

Participant

The research subjects are the students of semester VII of Mathematics Education Study Program University of PGRI Madiun and Madura University who are studying Field Experience Practice. In this study selected 5 of 105 mathematics pre-service teacher who are suspected of having good mathematical content knowledge that have cumulative grade index of more than 3.50 because the researcher could more explore the explanation. Teaching practice of math pre-service teacher for about five meetings (1.5 months) in the same class with students about 32-35 students. School for pre-service teachers teaching practice is public and private high school in Pamekasan, Sampang, and Madiun districts. Each student performs teaching with a learning implementation plan that is in line with the implementation of the 2013 curriculum.

Data Collection

The research data was obtained from observation, video recording, and interview. The video recording is used to document the learning practices undertaken by pre-service teachers. The focus of the research is the explanation of the pre-service teachers in the learning process takes place. The explanation made by pre-service teachers in explaining the learning of mathematics that can be an explanation of concepts and procedures of mathematics. The explanations made will be classified/categorized by the type/type of explanation that appears. Semi-structured interviews were conducted to find out the reasons for pre-service teachers in using explanatory forms in mathematics learning. Interview guides contain predefined key questions but are open-ended for researchers to have control over the topics for interviews (Creswell, 2012).

Data Analysis

Data analysis is done through three concurrent stages: (1) condensation data, (2) data presentation, and (3) drawing conclusion/verification (Miles, Huberman, & Saldana, 2014). Condensation of data refers to the process of selecting, focusing, simplifying, abstracting, and altering data that appears on observations, video recordings, and interviews. Presentation of data in this study is a collection of information that has been organized that allows the withdrawal of conclusions and actions. Next is an analytical activity that includes conclusion and verification. Data analysis in this research is done by reading again video transcript and

interview which then done triangulation so that can be concluded. Conclusions are still temporary and may change, so verification is required to obtain valid research results.

RESULTS AND DISCUSSION

The results of data analysis on this research are the explanations presented by the pre-service teacher during the practice of mathematics learning during the field experience in school practice. Data on the explanation presented by the pre-service teacher is obtained by recording the learning in the classroom using the handy recording recorder. Data recording of learning by student candidates for math teacher is taken as many as five times meeting. The number and types of explanations made by pre-service mathematics teachers are presented in Table 2.

Table 2. Number of Explanation Usage in Mathematics Learning

Students	Type of Explanation		
	Interpretative	Descriptive	Giving-Reason
S1	3	10	9
S2	2	12	8
S3	2	14	7
S4	4	13	6
S5	3	11	12
Total Number (%)	14 (12.1%)	60 (51.7%)	42 (36.2%)

Table 2 shows the most dominant type of explanation used by students of mathematics teacher candidates is the descriptive explanation (51.7%). Another type of explanation used is the reasoning (36.2%) and the least used is the interpretive explanation (12.1%). Next will be described each type of explanation submitted by the student candidate math teacher.

The most dominant type of explanation used by pre-service teachers is the descriptive explanation. This type of explanation is used by pre-service teachers to explain the completion of sample problems and solving problems that are considered difficult by students. In explaining the solution of this problem, the pre-service teacher explains the mathematical procedures as descriptive explanation in the following dialog.

Teacher: yes I explain it on the board because groups 4 and 5 have nothing to do.

Teacher: No. 1

Notice the following table $f(x)$ values.

x	$f(x)$
1	3
1,5	3,5
1,9	3,9
1,99	3,99
1,999	3,999

Teacher: what is the question?

Student: The value of $\lim_{x \rightarrow 2^-} f(x) = \dots$

Teacher: let's who can read $\lim_{x \rightarrow 2^-} f(x)$ correctly. If anyone can read I give a gift.

Student 1: limit x approaches $f(x)$ approaches 2.

Student 2: limit $f(x)$ close to 2 from left.

Teacher: yes it is not true all yes. So the true limit $f(x)$ where x is close to 2 from the left.

Teacher: So what do you understand about the limit what?

Student: approaching

Teacher: yes it is true that the value is only close. So let's look at this table, all the values of this x are close to 2 from the left, so for the value of x close to 2 from the left, then the value of f(x) is getting closer to how?

Student: approaching 4

Teacher: yes true, so the value of $\lim_{x \rightarrow 2^-} f(x) = 4$

The explanation above shows that the pre-service teacher explains how to determine the value of $\lim_{x \rightarrow 2^-} f(x)$. The pre-service teacher explains by using the f(x) value table. The table made by pre-service teachers consists of two columns where the first column is the value of x and the second column is the function value at point x. Before explaining the answer to the question, the pre-service teacher first asks the students' understanding of the limit. After the student candidate ensures the student's understanding that the limit is close, then explains to the student that from the table can be seen for the value of x which is close to 2 from the left (because the table is written number 1.999) the value of f(x) is closer to 4 (since table listed number 3.999). The explanation of this form gives students knowledge of how to determine the value of a limit of value if it is known that the value of x and f(x) in the question. However, the pre-service teacher only explains how to read the table and does not explain the steps of explaining how the limit value matches the limit concept.

When a researcher asks a mathematics pre-service teacher, she gives a reason for the form of explanation given in the following dialog.

Researcher: Why do you explain by using such tables?

Teacher: Because it is easy to understand students in obtaining the limit value which is a value close to a certain value, so I present it in tabular form.

Researcher: Why do you choose the numbers in the table?

Teacher: The number I chose is based on the sample in the book. Digits the numbers behind the comma I estimate itself is approaching the number 2.

Researcher: Why do the numbers you select on the table are only numbers less than 2?

Teacher: The numbers I choose, I sort from the top starting from the smallest to the bigger to the nearest 2. As my focus only explains the answer of the $\lim_{x \rightarrow 2^-} f(x)$ value is the limit value of f(x) for x close to 2 from the left, then I only present in table numbers that are less than 2 only.

From the interview results, it appears that pre-service teachers only focus on the final answer to the problem without considering the concept correctly in the process of completion. This form of explanation is a result-oriented explanation and does not provide information on why a particular step in the solution is needed which is a process-oriented form of explanation. The effectiveness of both types of explanation indicates that students who are learning with process-oriented explanations outweigh students who are studying with product-oriented explanations on application tests (Lachner & Nückles, 2016).

The mastery of the concept of function limits is influenced by the beliefs of each pre-service student (Szydlik, 2000). On the one hand, the perception of students about calculus is that a set of facts and procedures to be applied that they do not understand or appreciate the theory underlying those facts and procedures that make some arguments ineffective and mathematical arguments unconvincing; students holding this belief are left with an incomplete or contradictory limit model. On the other hand, students can see calculus as logical and consistent. This perception permits their

access to formal definitions, the effort to solve limit problems, and concept drawings free from significant internal inconsistencies. Students with internal confidence sources provide a coherent explanation of limit and are more likely than college students with external confidence sources to hold static concepts about the limit (Szydlik, 2000).

In addition, more importantly, when students first enter university and from the beginning have brought misunderstandings or lack of deep understanding, it is difficult to restore. While trying to correct the misconception, students will often break away, because their beliefs and knowledge are challenged, which causes their thinking to be disturbed. It is therefore important to continually reinforce prior concepts in new situation, assisting students to build a relations between mathematical ideas (Bardini, Pierce, Vincent, & King, 2014).

Furthermore, another descriptive explanation form is used by the pre-service mathematics teacher to explain how the steps or problem-solving procedures are presented in the following dialog.

Teacher: Let me give you an example

Determine the rate of function change $f(x) = 3x^2 - 4x$ di $x=2$.

Teacher: Asked the rate of change function, here there is a formula.

Teacher: We try together.

Teacher and Student: $f(x) = 3x^2 - 4x$

Teacher: the formula there is $f(x + h)$

Student: yes.

Teacher: Means $f(x + h) = \dots$ Try you (point to one of the students), $f(x + h)$ what is it?

Student: Mom, why look for $f(x + h)$

Teacher: we see the formula, it assumes that there is $f(x + h)$, we point it as in the formula.

Student: ooo.

Teacher: Let's $f(x + h)$ how much?

Teacher: kan $f(x) = 3x^2 - 4x$ maka $f(x+h) = 3(x+h)^2 - 4(x+h)$.

Teacher: we change that x to $x + h$

Student: Ooo...

Teacher: put the form directly into it.

$$\begin{aligned}
 f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{(3(x+h)^2 - 4(x+h)) - (3x^2 - 4x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{(3(x^2 + 2xh + h^2) - 4(x+h)) - (3x^2 - 4x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{3x^2 + 6xh + 3h^2 - 4x - 4h - 3x^2 + 4x}{h} \\
 &= \lim_{h \rightarrow 0} \frac{6xh + 3h^2 - 4h}{h} \\
 &= \lim_{h \rightarrow 0} \frac{h(6x + 3h - 4)}{h} \\
 &= \lim_{h \rightarrow 0} 6x + 3h - 4 \\
 &= 6x - 4
 \end{aligned} \tag{1}$$

Obtained $f'(x) = 6x - 4$. Thus $f'(2) = 6(2) - 4 = 12 - 4 = 8$.

Teacher: Up here understand?

Student: Understood.

Another descriptive explanation form by the mathematics teacher candidate is to explain the rate of function change $f(x) = 3x^2 - 4x$ at $x = 2$. In explaining the rate of change of the function, the pre-service teacher explains to the student if to solve the problem using the derived formula $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$.

The pre-service teacher explains the steps of using the formula by first finding the value of $f(x+h)$. Having obtained the value of $f(x+h)$ is then entered in the formula using the usual algebraic operation which then obtained the value of the limit to h close to 0. The process of determining the value of $f(x)$ which continues to find the value of $f(x+h)$ is the first step in mathematical thinking namely specializing, which begins solving the problem by working on the small part first (Stacey, 2009). The problem-solving procedure described by the pre-service teacher also uses analog thinking, working on problems based on similarities of nature and procedures. Thinking analogies is a very important method of thinking to build perspectives and find solutions (Isoda & Katagiri, 2012).

The type of explanation often used by pre-service teachers in explaining after the descriptive explanation is the giving-reason explanation in the following dialog.

$$\begin{aligned} \text{Student 1: } 3f'(11) &= 2 + \frac{3}{4} \\ 3f'(11) &= \frac{8+3}{4} \\ 3f'(11) &= \frac{11}{4} \end{aligned} \tag{2}$$

Student 2: Mam, why this $2 + \frac{3}{4}$ can be like this $\frac{8+3}{4}$?

Teacher: $2 + \frac{3}{4}$, because it is a sum of fractions, it must be equalized denominator, so $\frac{2}{1}$ should be equalized denominator to 4.

So $\frac{8}{4} + \frac{3}{4} = \frac{11}{4}$. Understood Toriq?

Student 2: Yes understand Mam...

The explanation of reasoning is an explanation of the question of why something can happen. This form of explanation usually depends on the questions that is asked by students to pre-service teachers. One type of explanation of the giving-reason by the pre-service teacher is an explanation that arises because the student's question is why the sum of integers with $2 + \frac{3}{4}$ can be $\frac{8+3}{4}$. In the explaining the student's question, the pre-service teacher reminded students of the nature/principle used in addition to fractions. In that explanation, the pre-service teacher changed the number 2 into fraction $\frac{2}{1}$, which then summed up with fraction $\frac{3}{4}$. Through the principle of the sum of the denominations having different denominators, the denominator of the two denominations must first be equalized 4. With the denominator 4 then the numerator of the first fraction becomes 8 so that the number 2 equals $\frac{8}{4}$ and finally obtained $\frac{8}{4} + \frac{3}{4}$ being $\frac{8+3}{4}$.

The student's questions related to the operation of the fractions are in line with Erlinda & Surya (2017) stating that for some fractional students is one concept that is considered difficult to master. In answering the student's question, the pre-service teacher can give a good explanation. This is because the pre-service teacher has good knowledge about the addition operation in fractional numbers. The explanation will be different if the student's question is about the division operation on the fraction. In explaining fractional operations, it was identified that the difficulties experienced by pre-service high school teachers were at the time of explaining the fractional divisions (Li & Smith, 2007).

The form of explanation for other reasons that appear in the learning is on the derived material function as in the following dialog.

Teacher: we change the first power form. For example, I give you an example.

$f(x) = \sqrt{x+2}$ means $f(x)$ become $(x+2)^{\frac{1}{2}}$. Now $f(x)$ we can derived, enter into the derivative

properties of u^n . So to look for derivatives using the properties of u^n .

$$f(x) = \sqrt{x+2}$$

$$f(x) = (x+2)^{\frac{1}{2}}$$

$$f'(x) = \frac{1}{2}(x+2)^{\frac{1}{2}-1}(1)$$

$$f'(x) = \frac{1}{2}(x+2)^{-\frac{1}{2}}$$

$$f'(x) = \frac{1}{2\sqrt{x+2}}$$

Teacher: Up here understand?

Student: Understood.

Student: who is it, why multiplied by 1.

Teacher: this one? Why multiplied by 1?

Student: yes

Teacher: why is this 1, there is a formula, and this can be transformed into this trait (u^n). If there is a function in this form (u^n), then its derivative uses this formula (derived formulas). Which yesterday was mother taught? If this matter is the power of $\frac{1}{2}$ means $\frac{1}{2}(x+2)$ the power of half subtracted by 1.

$\frac{1}{2} - 1 = \frac{1}{2} - \frac{2}{2} = -\frac{1}{2}$. Now u derived. $u = (x+2)$ if $(x+2)$ is derived then $u' = 1$. Understand?

Student: yes.

Teacher: if you do not understand, you can ask.

Teacher: So multiplied by 1

$$f'(x) = \frac{1}{2}(x+2)^{\frac{1}{2}-1}(1)$$

$$f'(x) = \frac{1}{2}(x+2)^{-\frac{1}{2}}$$

Teacher: this is a negative power, we make a positive power. Using the nature of the number power.

The explanation type of the giving reason above arises because the question of the student asking why the 3rd step is $\frac{1}{2}(x+2)^{\frac{1}{2}-1}$ is multiplied by 1. The pre-service teacher gives the reason for the question by basing on rules/derived function principles. If there is a function in this form (u^n), then its derivative uses this formula (derived formulas). If this matter is the power of $\frac{1}{2}$ means $\frac{1}{2}(x+2)$ the power of half is subtracted by by 1. Next $\frac{1}{2} - 1 = \frac{1}{2} - \frac{2}{2} = -\frac{1}{2}$. Now u is derived if $u = (x+2)$ if is derived then $u' = 1$. The explanation of the giving reason is an analogy according to the principle of a derivative function. The pre-service teachers in this case also use analog thinking that based on the similarity of nature/principle to obtain the solution of the problem (Isoda & Katagiri, 2012).

Student: It's the same pack of compound interest as compound interest with the fractional interest.

Teacher: Oh that's the difference if the compound interest is the amount of deposit in question exactly match the interest given per period. If those with a fractional interest is usually the time of determining the amount of savings is not exactly even with interest given per period. For example, if the interest is given per year when asked the amount of savings usually do not fit per year, for example, can be 4 years 6 months.

The interpretative explanation is an explanation of what questions, so it can be a definition and definition of a concept. This interpretative explanation student form appears on the student's question as

to what is the distinction between compounded interest and compound interest during the fractional interest. Pre-service teacher gives an explanation that if on the question, the compound interest is the number of deposits in question exactly the interest given per period. While compound interest with a fractional interest is usually the time of determining the amount of savings is not exactly even with interest given per period. For example, if the interest given per year when asked the number of deposits is usually not fitting per year for example 4 years over 6 months. Explanations made by pre-service teachers of this teacher are appropriate because it can understand students about the understanding of compound interest and compound interest with the interest period. In teaching the concept, the main thing a teacher should do is give the student a definition of the concept or guide the student to find out the definition of the concept followed by giving or requesting an example of the concept (Mohr-Schroeder, Ronau, Peters, Lee, & Bush, 2017).

In addition to explaining the concept of mathematics, this type of interpretive explanation can also be used to explain the facts of mathematics which are conventions or agreements that can be presented in a symbolic form, commonly understood by mathematical users (Gagne, 1985). This form of explanation happens when the pre-service teacher explains to the student about the symbol f' which is the first derivative of function f . This explanation is given because the student has not received any material about the derivative of the function in the previous mathematics lesson.

CONCLUSION

The use of the type of explanation in the most dominant mathematical learning is the type of descriptive explanation. The next type of explanation that is not very often used is a reasoning explanation. While the kind of explanation most rarely used is the type of interpretive explanation. In learning mathematics, the type of descriptive explanation used by pre-service teachers to explain how the procedure and the steps of solving problems or problems of mathematics to students. This type of explanation arises when the teacher gives an example of problem-solving and when the student is having trouble with a given mathematical problem. The use of this type of explanation usually appears in the core activities of learning.

The next type of explanation is a reasoning explanation. Mathematics pre-service teacher use this form of explanation in providing answers to why certain procedures and steps of settlement can be selected based on existing rules/principles. Also, this type of explanation can be used by pre-service teachers in giving reasons why a principle in mathematics can occur. The last type of explanation rarely used by mathematics teacher candidates is the kind of interpretive explanation. This type of explanation is used to explain what the notion and definition of mathematical concepts are. Pre-service teachers can also use this form of explanation in explaining what questions about mathematical facts.

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